EE 230
Lecture 43

Data Converters

Review from Last Time:

Amplitude Quantization

Unwanted signals in the output of a system are called noise.

Distortion

- Smooth nonlinearities
- Frequency attenuation
- Large Abrupt Nonlinearities

Signals coming from other sources

- Movement of carriers in devices
- Interference from electrical coupling
- Interference from radiating sources

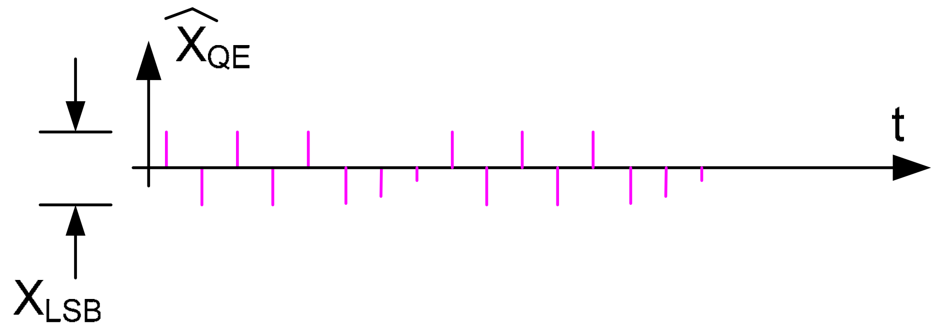
Undesired outputs inherent in the data conversion process itself

Review from Last Time:

Characterization of Quantization Noise

Sinusoidal excitation

- Consider an ADC (clocked)



Theorem: If n(t) is a random process, then $V_{RMS} \cong \sqrt{\sigma^2 + \mu^2}$

provided that the RMS value is measured over a large interval where the parameters σ and μ are the standard deviation and the mean of <n(kT)>

This theorem can thus be represented as

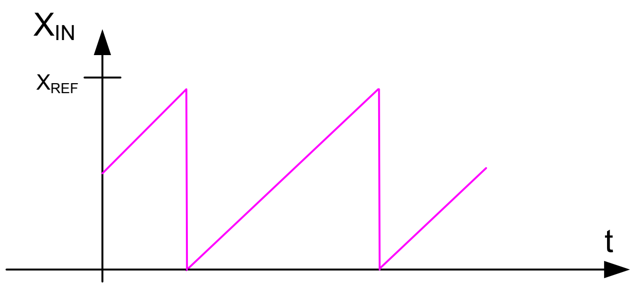
$$V_{RMS} \cong \sqrt{\frac{1}{T_L}\int_{t_1}^{t_1+T_L} n^2(t)\,dt} \cong \sqrt{\sigma^2 + \mu^2}$$

where T is the sampling interval and T_L is a large interval

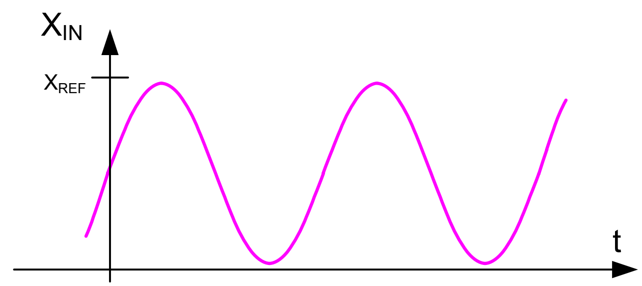
Review from Last Time:

Characterization of Quantization Noise

Saw tooth excitation



Sinusoidal excitation



$$X_{Q\text{-}RMS} \cong \frac{X_{LSB}}{\sqrt{12}}$$

Saw tooth:

$$SNR = 2^n$$

$$SNR_{dB} = 6.02n$$

Sinusoidal:

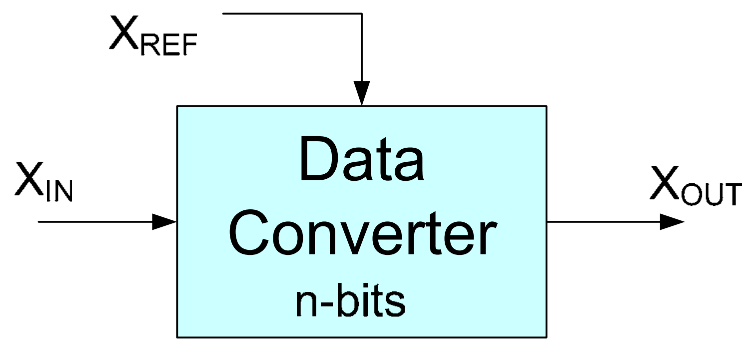
$$SNR = 1.225 \bullet 2^n$$

$$SNR_{dB} = 6.02n + 1.76$$

Although derived for an ADC, same expressions apply for DAC
SNR for saw tooth and for triangle excitations are the same
SNR for sinusoidal excitation larger than that for saw tooth by 1.76dB
SNR will decrease if input is not full-scale
Equivalent Number of Bits (ENOB) often given relative to quantization noise SNR_{dB}
Remember – quantization noise is inherent in an ideal data converter!

Review from Last Time:

Equivalent Number of Bits (ENOB)



$$\text{ENOB} = \frac{\text{SNR - 1.76}}{6.02} \qquad \text{ENOB} = \frac{\text{SNDR - 1.76}}{6.02}$$

These definitions of ENOB are based upon noise or noise and distortion

Some other definitions of ENOB are used as well – e.g. if one is only interested in distortion, an ENOB based upon distortion can be defined.

ENOB is useful for determining whether the number of bits really being specified is really useful

Engineering Issues for Using Data Converters

1. Inherent with Data Conversion Process

- Amplitude Quantization
- Time Quantization

(Present even with Ideal Data Converters)

2. Nonideal Components

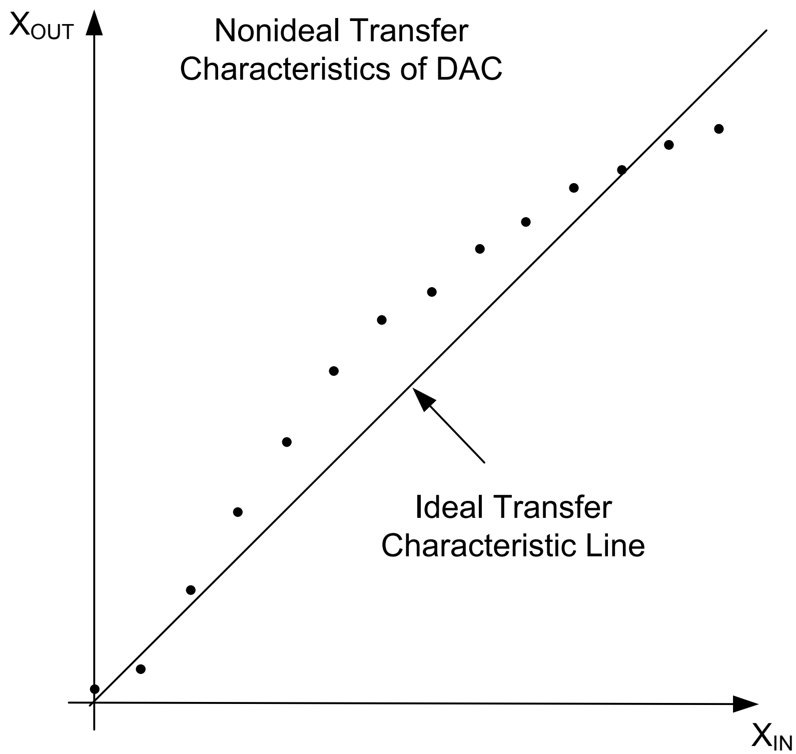
- Uneven steps
- Offsets
- Gain errors
- Response Time
- Noise

(Present to some degree in all physical Data Converters)

How do these issues ultimately impact performance ?

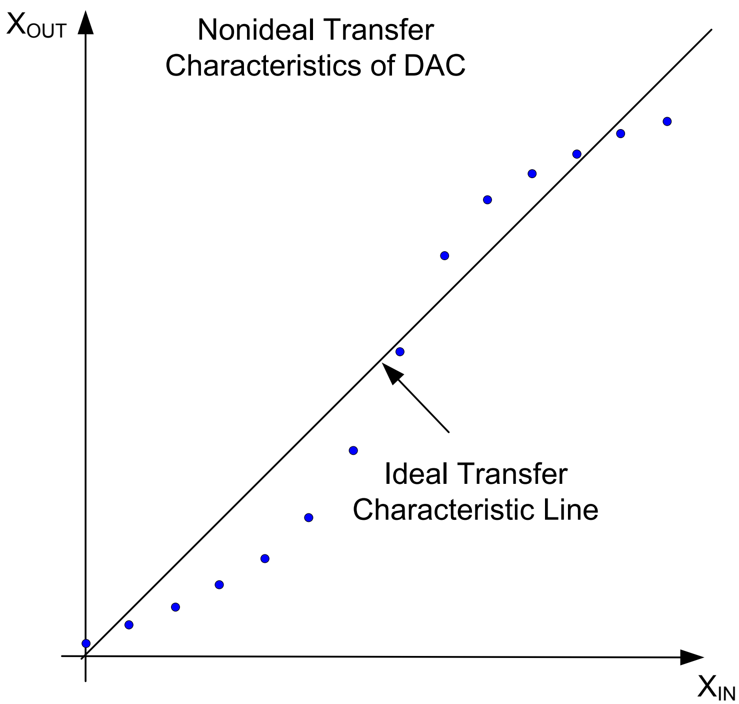
Nonideal Transfer Characteristics

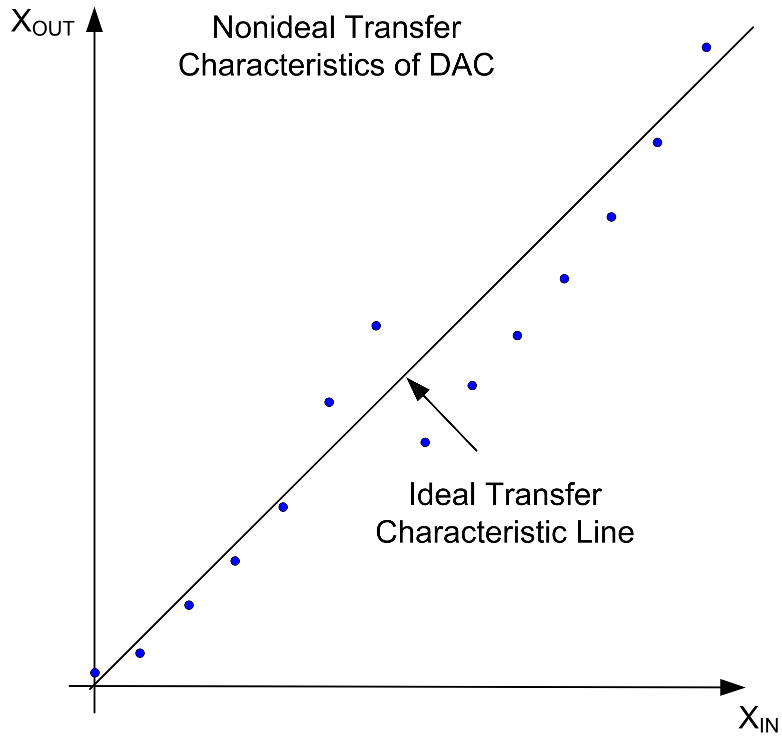
Uneven Steps



Nonideal Transfer Characteristics

Uneven Steps

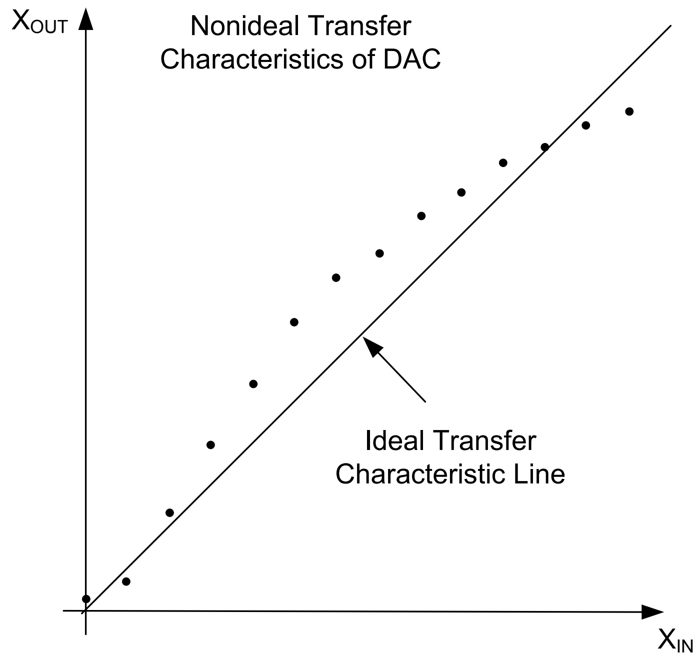




Actual transfer characteristics can vary considerably from one device to another

Nonideal Transfer Characteristics

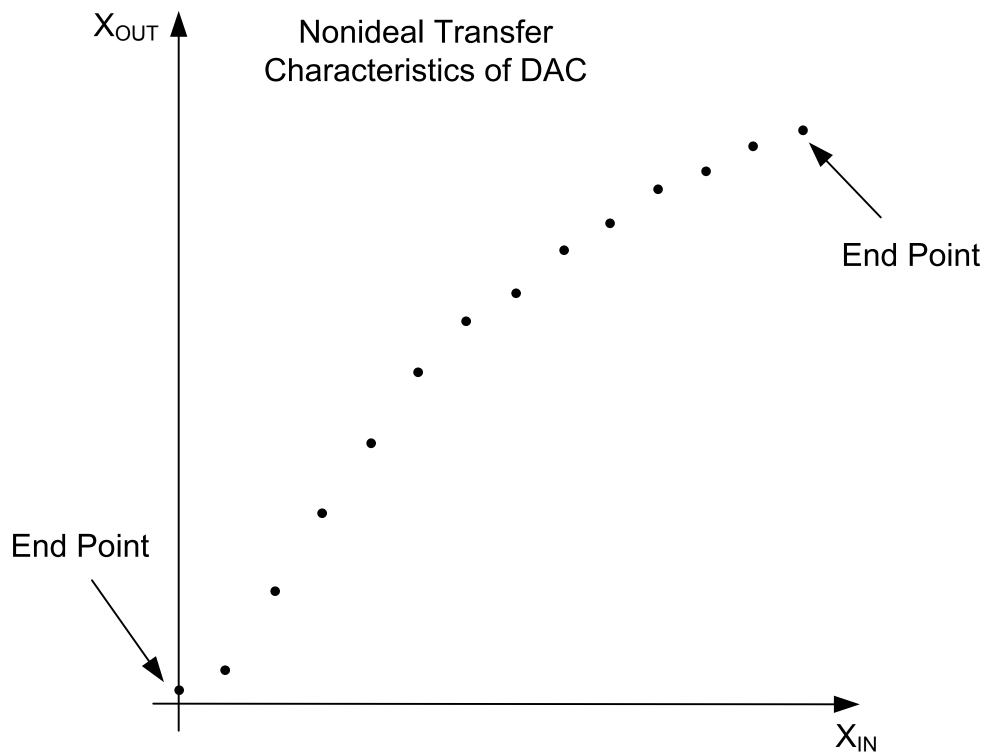
Uneven Steps



This is termed a nonlinearity in the data converter

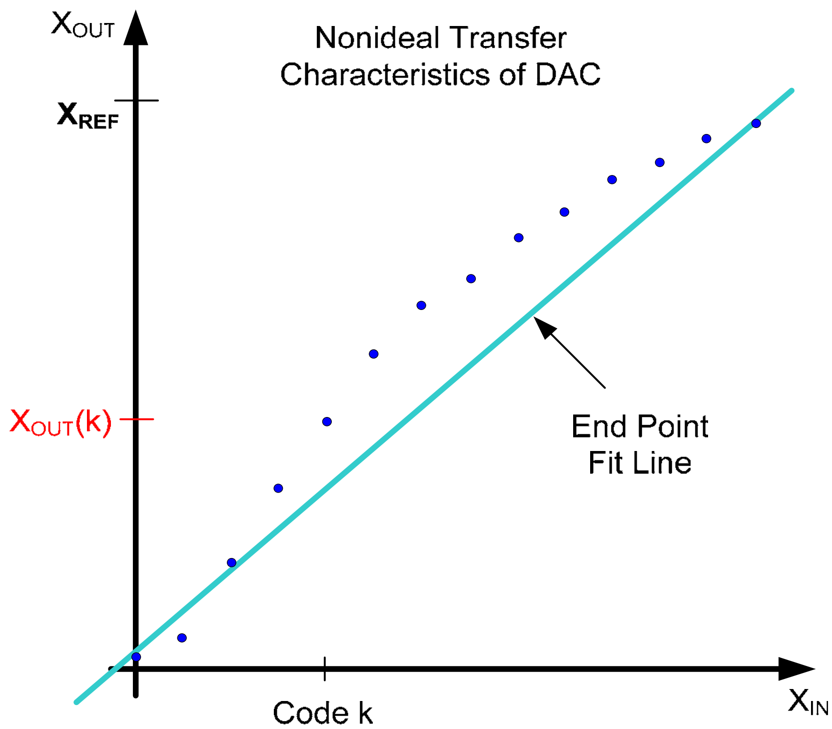
Linearity metrics (specifications) include INL, DNL, THD and SFDR

Characterization of Nonlinearities



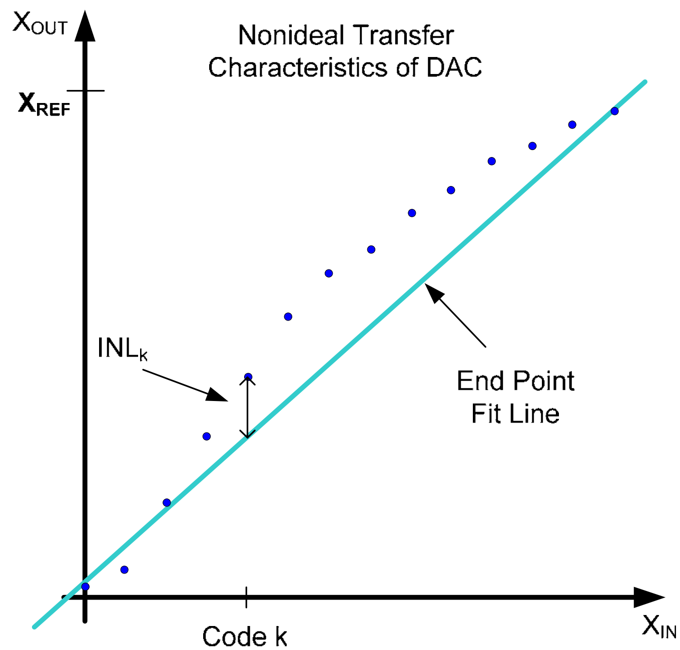
End points are the outputs at the two extreme Boolean inputs

Characterization of Nonlinearities



End point fit line

Characterization of Nonlinearities



Linearity metrics:

- INL
- DNL
- THD
- SFDR

Integral Nonlinearity (INL)

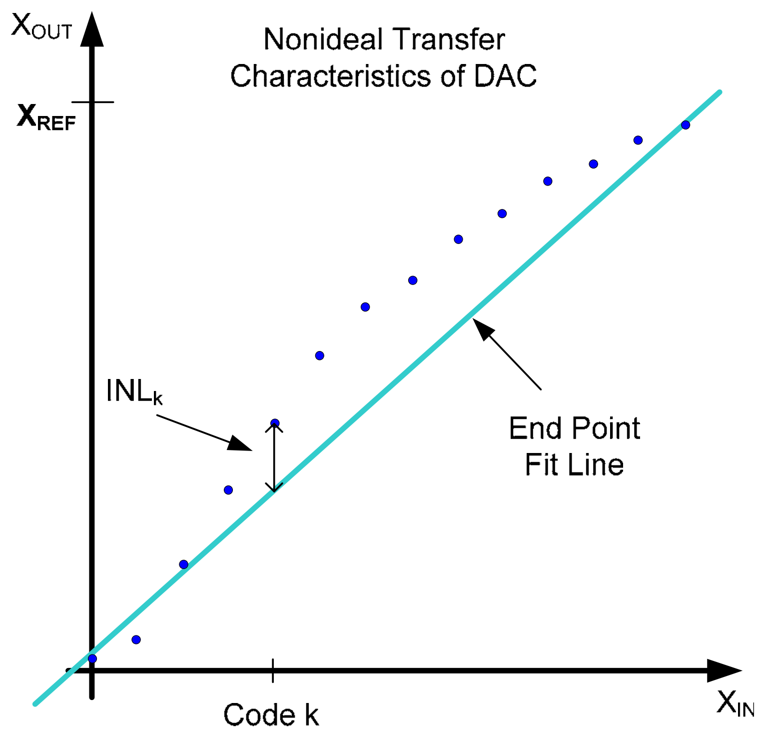
Measure of worst-case deviation from linear

Define the INL at any input code k by:

$$INL_k = X_{OUT}(k) - X_{FIT}(k)$$

Integral Nonlinearity (INL)

Linearity metrics:
- INL
- DNL
- THD
- SFDR



Define the INL by:

$$\mathrm{INL} = \max_{1 \le k \le N} \left\{ \left| \mathrm{INL}_k \right| \right\}$$

Integral Nonlinearity (INL)

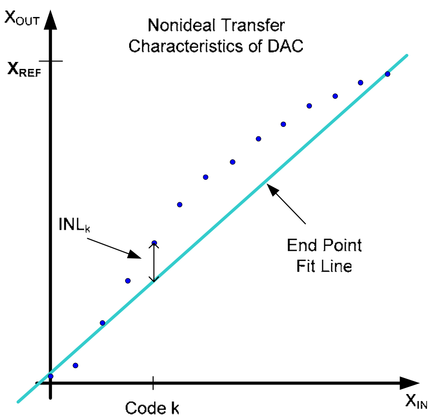
$$\mathrm{INL} = \max_{1 \le k \le N} \{ |\mathrm{INL}_k| \}$$

Often expressed in LSB:

$$\mathrm{INL}_{LSB} = \frac{\mathrm{INL}}{X_{LSBF}}$$

where

$$X_{LSBF} = \frac{X_{OUT}(N) - X_{OUT}(1)}{N-1}$$



Linearity metrics:

- INL
- DNL
- THD
- SFDR

Integral Nonlinearity (INL)

$$\text{INL} = \max_{1 \le k \le N} \left\{ \left| \text{INL}_k \right| \right\}$$

Often expressed in LSB:

$$\text{INL}_{\text{LSB}} = \frac{\text{INL}}{X_{\text{LSBF}}}$$

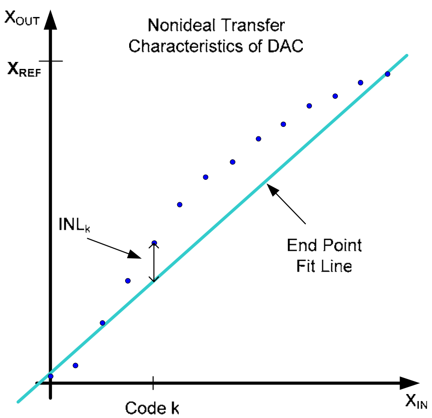
where

$$X_{\text{LSBF}} = \frac{X_{\text{OUT}}(N) - X_{\text{OUT}}(1)}{N-1}$$

but

$$X_{\text{LSBF}} = \frac{X_{\text{OUT}}(N) - X_{\text{OUT}}(1)}{N-1} \cong \frac{X_{\text{REF}}}{2^n}$$

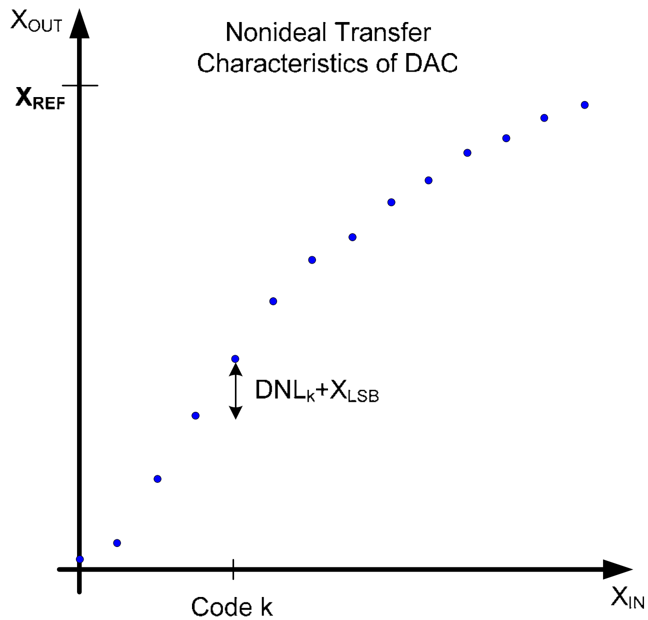
$$\text{INL}_{\text{LSB}} \cong 2^n \frac{\text{INL}}{X_{\text{REF}}}$$



Linearity metrics:

- INL
- DNL
- THD
- SFDR

Characterization of Nonlinearities



Linearity metrics:
- INL
- DNL
- THD
- SFDR

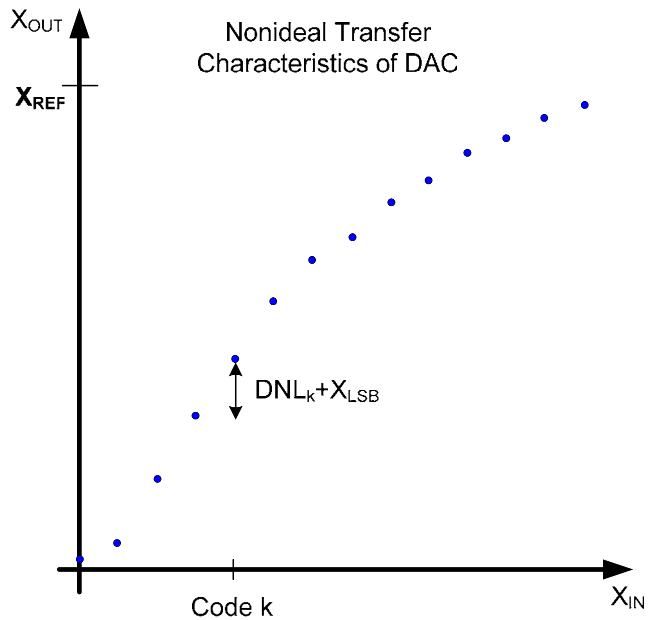
Differential Nonlinearity (DNL)

Measure of worst-case resolving capabilities

Define the DNL at any input code k by:

$$DNL_k = X_{OUT}(k) - X_{OUT}(k-1) - X_{LSBF} \cong X_{OUT}(k) - X_{OUT}(k-1) - X_{LSB}$$

Differential Nonlinearity (DNL)



Linearity metrics:

- INL
- DNL
- THD
- SFDR

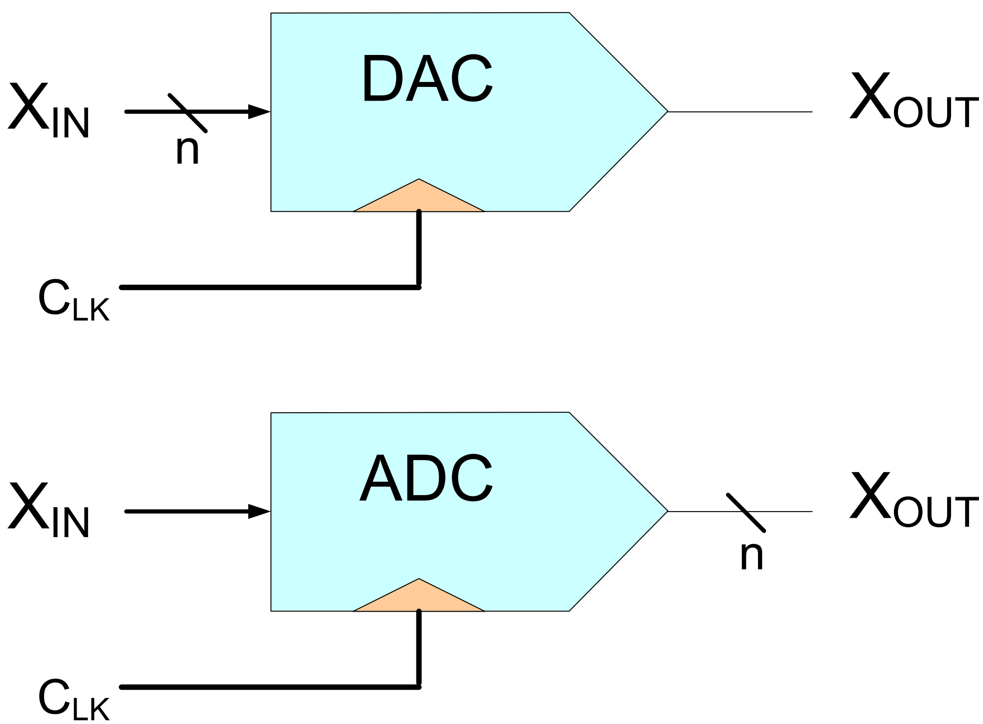
$$DNL_k \cong X_{OUT}(k)\text{-}X_{OUT}\left(k\text{-}1\right)\text{-}X_{LSB}$$

$$DNL = \max_{1<k\le N}\left\{\left|DNL_k\right|\right\}$$

Often expressed in LSB

$$DNL_{LSB} = \frac{DNL}{X_{LSBF}} \quad \longrightarrow \quad DNL_{LSB} \cong 2^n \frac{DNL}{X_{REF}}$$

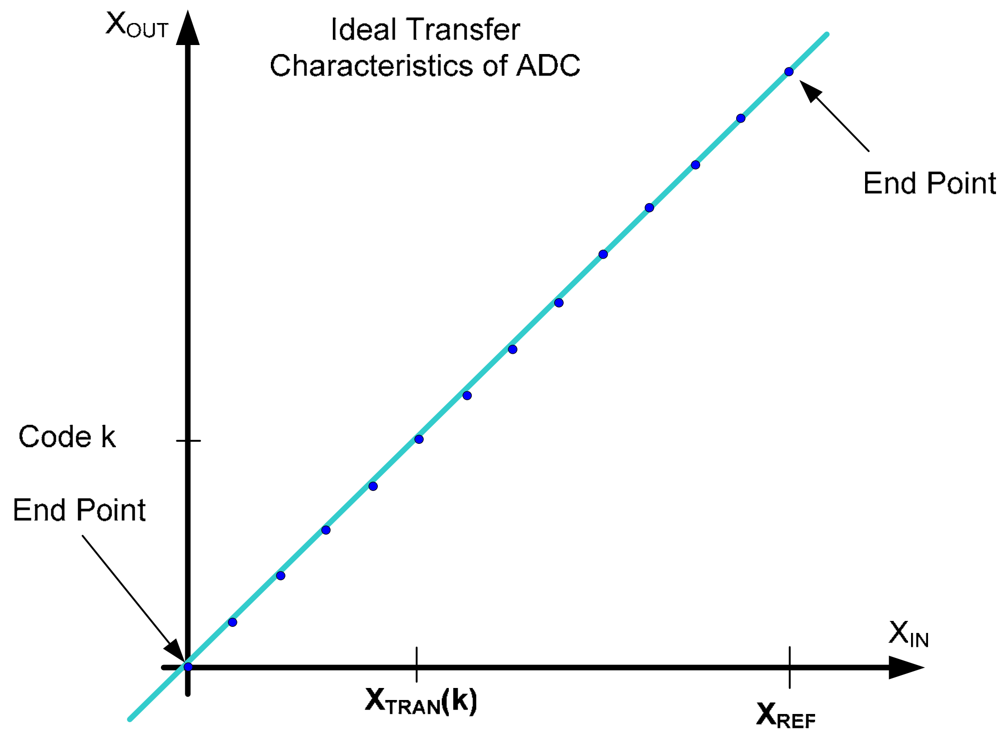
Characterization of Nonlinearities



Linearity metrics:
INL
DNL
THD
SFDR

Linearity Metrics for ADC and DAC are Analogous to Each Other

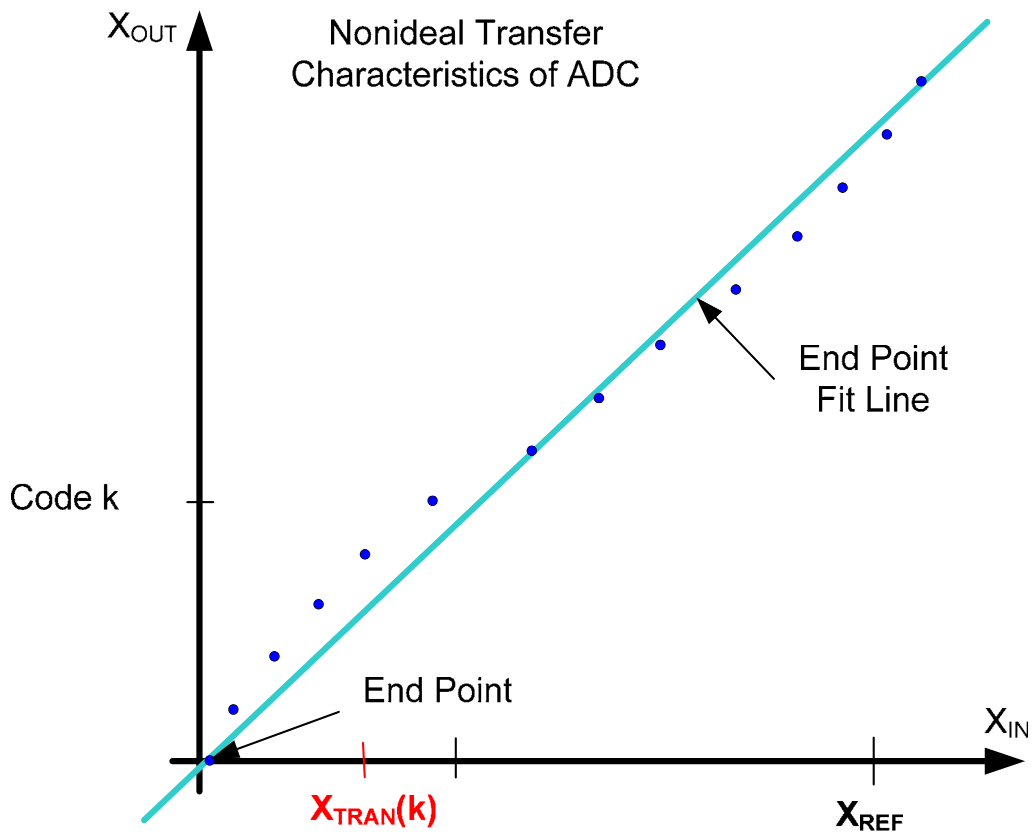
Integral Nonlinearity (INL)



Linearity metrics:

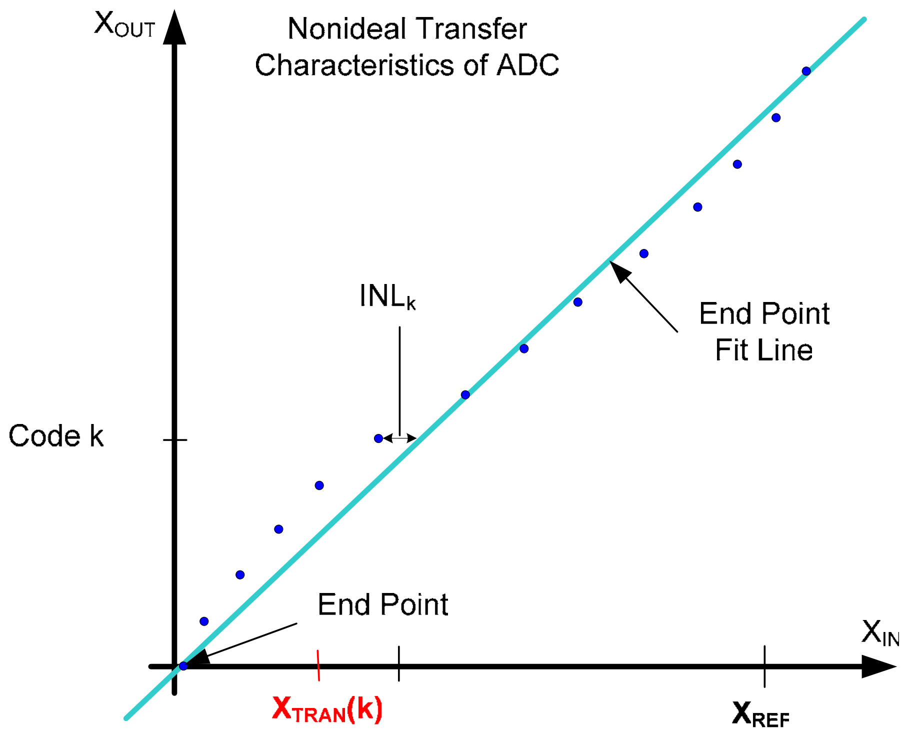
- INL
- DNL
- THD
- SFDR

Integral Nonlinearity (INL)



Linearity metrics:
- INL
- DNL
- THD
- SFDR

Integral Nonlinearity (INL)



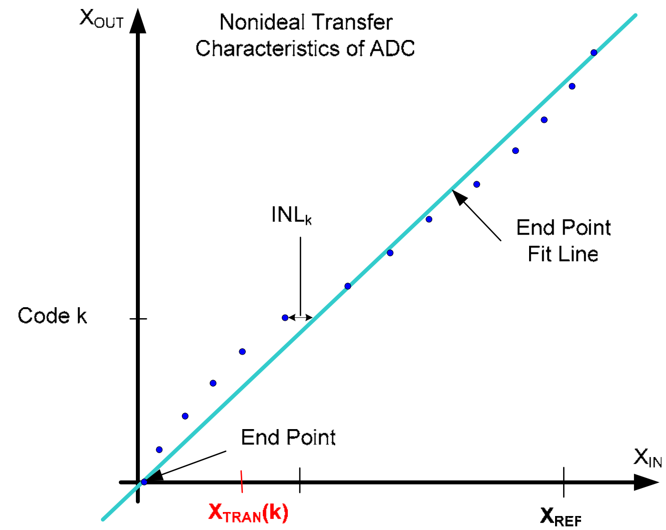
Linearity metrics:

- INL
- DNL
- THD
- SFDR

$$INL_k = X_{TRAN}(k) - X_{FIT}(k)$$

$$X_{FIT}(k) = X_{TRAN}(1) + \left(\frac{k-1}{N-2}\right)\frac{X_{TRAN}(N-1) - X_{TRAN}(1)}{N-2}$$

Integral Nonlinearity (INL)



Linearity metrics:
- INL
- DNL
- THD
- SFDR

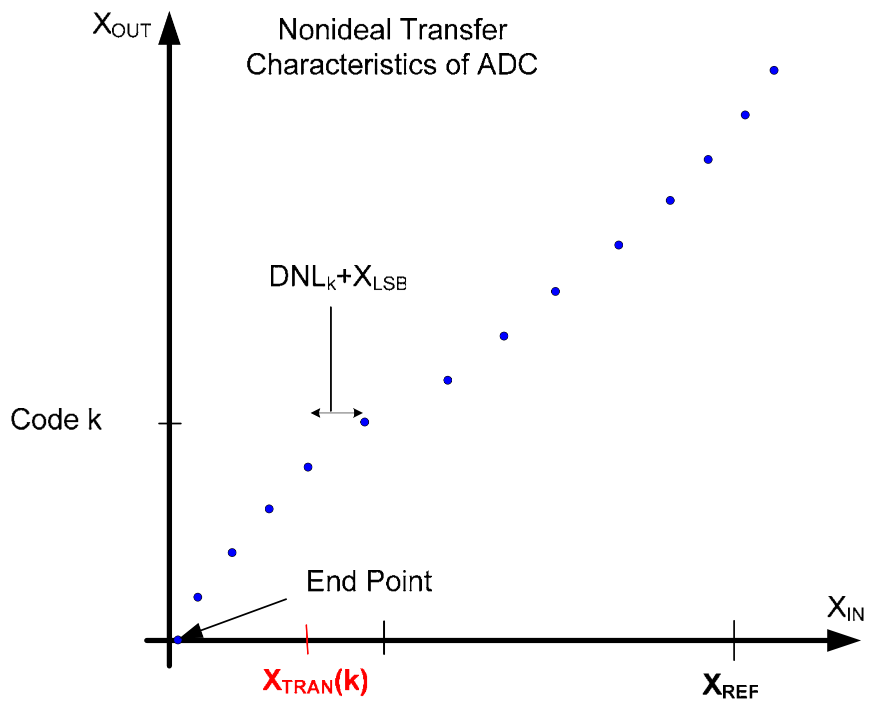
$$INL_k = X_{TRAN}(k) - X_{FIT}(k)$$

$$INL = \max_{1 \le k \le N} \{ |INL_k| \}$$

$$INL_{LSB} = \frac{INL}{X_{LSBF}}$$

$$INL_{LSB} \cong 2^n \frac{INL}{X_{REF}}$$

Differential Nonlinearity (DNL)



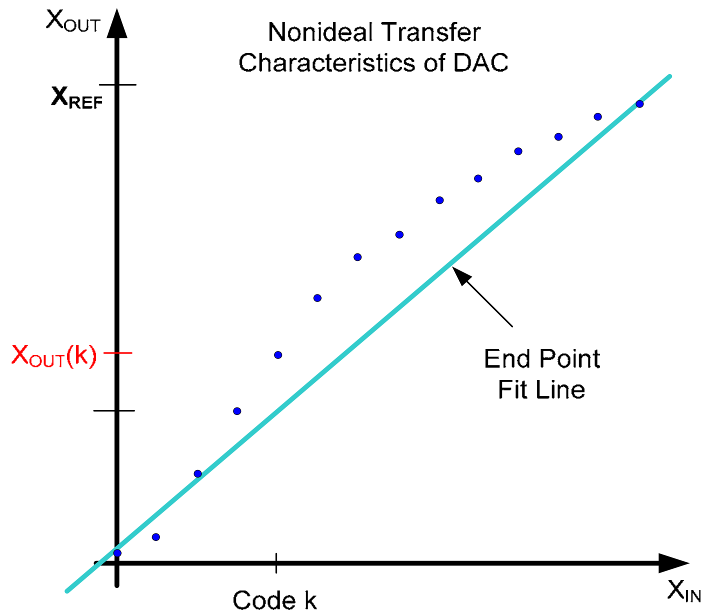
Linearity metrics:

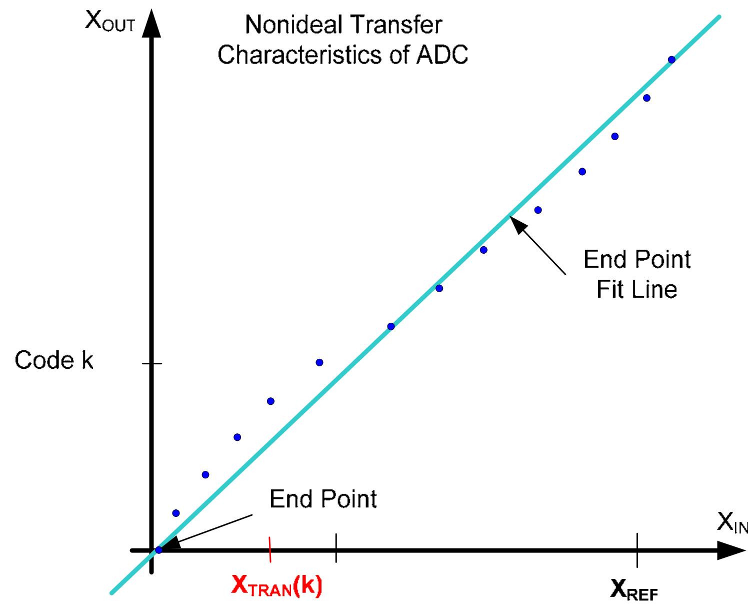
- INL
- DNL
- THD
- SFDR

$$DNL_k \cong X_{TRANS}(k) - X_{TRANS}(k-1) - X_{LSB}$$

$$DNL = \max_{1<k\leq N}\left\{\left|DNL_k\right|\right\}$$

Equivalent Number of Bits -ENOB (based upon linearity)

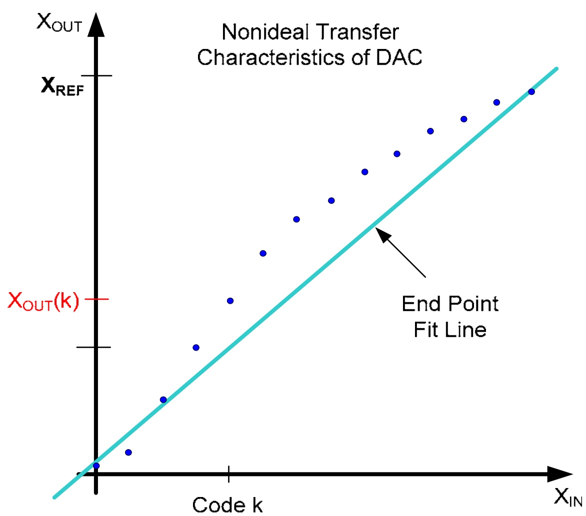


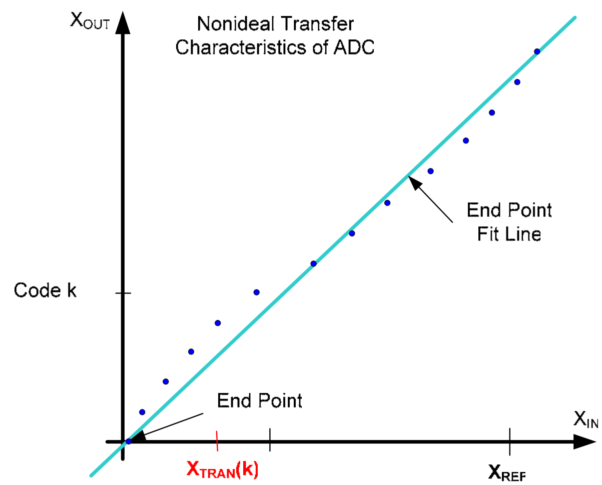


Generally expect INL to be less than ½ LSB

If INL larger than ½ LSB, effective resolution is less than specified resolution

Equivalent Number of Bits -ENOB (based upon linearity)



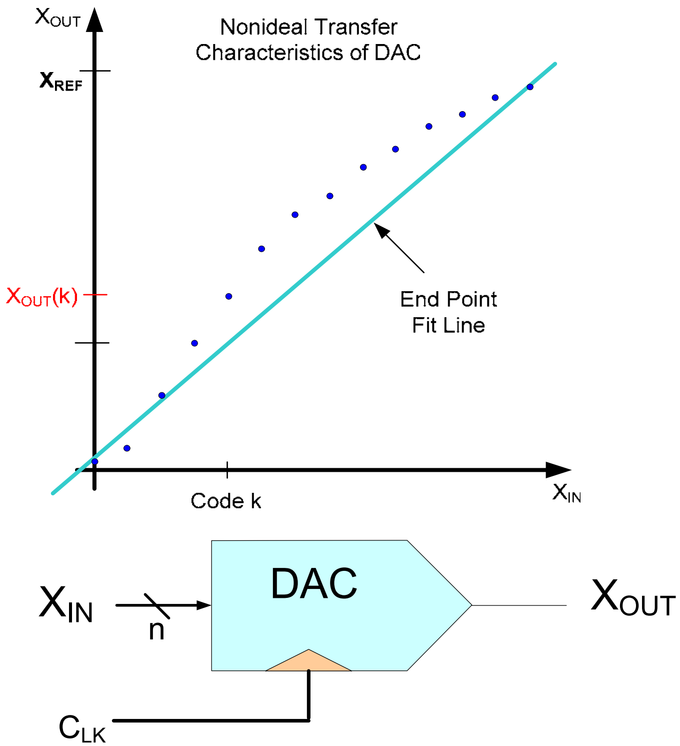


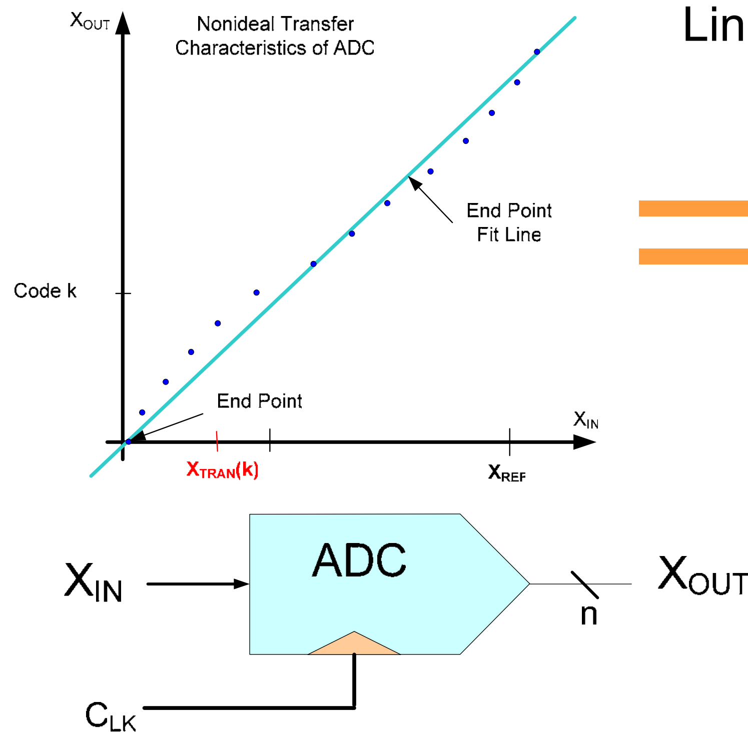
If v is the INL in LSB

$$\text{ENOB} = \text{n-}1 - \frac{\log_{10} \nu}{\log_{10} 2}$$

v	res
0.5	n
1	n-1
2	n-2
4	n-3
8	n-4
16	n-5

Spectral Characterization





Linearity metrics:
- INL
- DNL
- THD
- SFDR

$$X_{IN} = X_M \sin(\omega t + \theta)$$

If nonlinearities present, X_{OUT} given by

$$X_{OUT} = A_0 + A_1 \sin(\omega t + \theta + \gamma_1) + \sum_{k=2}^{\infty} A_k \sin(k\omega t + \theta + \gamma_k)$$

Spectral Characterization

$$X_{IN} = X_M \sin(\omega t + \theta)$$

$$X_{OUT} = A_0 + A_1 \sin(\omega t + \theta + \gamma_1) + \sum_{k=2}^{\infty} A_k \sin(k\omega t + \theta + \gamma_k)$$

A_k, k>1 are all spectral distortion components

Generally only first few terms are large enough to represent significant distortion

$$THD = \frac{\sum_{k=2}^{\infty} A_k^2}{A_1^2}$$

$$THD_{dB} = 10\log_{10}\left(\frac{\sum_{k=2}^{\infty} A_k^2}{A_1^2}\right)$$

$$SFDR = \frac{|A_1|}{\max_{1<k}\{|A_k|\}}$$

$$SFDR_{dB} = 20\log_{10}\left(\frac{|A_1|}{\max_{1<k}\{|A_k|\}}\right)$$

Spectral Characterization

$$X_{IN} = X_M \sin(\omega t + \theta)$$

$$X_{OUT} = A_0 + A_1 \sin(\omega t + \theta + \gamma_1) + \sum_{k=2}^{\infty} A_k \sin(k\omega t + \theta + \gamma_k)$$

Generally X_M is chosen nearly full-scale and input is offset by $X_{REF}/2$

$$X_{IN} = \frac{X_{REF}}{2} + \left(\frac{X_{REF}}{2} - \varepsilon\right) \sin(\omega t + \theta)$$

Direct measurement of A_k terms not feasible

A_k generally calculated from a <u>large</u> number of samples of $X_{OUT}(t)$

Spectral Characterization

$$X_{IN} = X_M \sin(\omega t + \theta)$$

$$X_{OUT} = A_0 + A_1 \sin(\omega t + \theta + \gamma_1) + \sum_{k=2}^{\infty} A_k \sin(k\omega t + \theta + \gamma_k)$$

Key theorem useful for spectral characterization

Theorem: If a periodic signal x(t) with period T=1/f is band-limited to frequency hf and if the signal is sampled N times over an integral number of periods, N_P, then

$$\left|A_m\right| = \frac{2}{N}\left|X(mN_P + 1)\right| \qquad \textbf{for } 0 \le m \le h\text{-}1$$

where $\left\langle X(k) \right\rangle_{k=1}^{N-1}$ **is the DFT of the sampled sequence** $\left\langle x(kT_S) \right\rangle_{k=1}^{N-1}$ **where T_S is the sampling period.**

$$T_S = \frac{T \bullet N_P}{N}$$

Spectral Characterization

$$X_{IN} = X_M \sin(\omega t + \theta)$$

$$X_{OUT} = A_0 + A_1 \sin(\omega t + \theta + \gamma_1) + \sum_{k=2}^{\infty} A_k \sin(k\omega t + \theta + \gamma_k)$$

Key theorem useful for spectral characterization

Theorem: If a periodic signal x(t) with period T=1/f is band-limited to frequency hf and if the signal is sampled N times over an integral number of periods, N_P, then

$$|A_m| = \frac{2}{N} \left| X(mN_P + 1) \right| \qquad \text{for } 0 \le m \le h-1$$

where $\langle X(k) \rangle_{k=1}^{N-1}$ **is the DFT of the sampled sequence** $\langle x(kT_S) \rangle_{k=1}^{N-1}$ **where T_S is the sampling period.**

- This theorem is usually not stated although widely used
- Often this theorem is misunderstood or misused
- If hypothesis not exactly satisfied, major problems with trying to use this theorem

Spectral Characterization

$$X_{IN}=X_M \sin(\omega t+\theta)$$

$$X_{OUT}=A_0+A_1\sin(\omega t+\theta+\gamma_1)+\sum_{k=2}^{\infty}A_k\sin(k\omega t+\theta+\gamma_k)$$

Key theorem useful for spectral characterization

Theorem: If a periodic signal x(t) with period T=1/f is band-limited to frequency hf and if the signal is sampled N times over an integral number of periods, N_P, then

$$|A_m|=\frac{2}{N}\left|X(mN_P+1)\right| \qquad \text{for } 0 \le m \le h\text{-}1$$

where $\left\langle X(k)\right\rangle_{k=1}^{N-1}$ **is the DFT of the sampled sequence** $\left\langle x(kT_S)\right\rangle_{k=1}^{N-1}$ **where T_S is the sampling period.**

- This theorem is usually not stated although widely used
- Often this theorem is misunderstood or misused
- If hypothesis not exactly satisfied, major problems with trying to use this theorem

Spectral Characterization

Key theorem useful for spectral characterization

Theorem: If a periodic signal x(t) with period T=1/f is band-limited to frequency hf and if the signal is sampled N times over an integral number of periods, N_P, then

$$\left|A_m\right| = \frac{2}{N}\left|X\left(mN_P+1\right)\right| \qquad \textbf{for } 0 \le m \le h\text{-}1$$

where $\left\langle X\left(k\right)\right\rangle_{k=1}^{N-1}$ **is the DFT of the sampled sequence** $\left\langle x\left(kT_S\right)\right\rangle_{k=1}^{N-1}$ **where T_S is the sampling period.**

- Usually N_p is a prime number (e.g. 11, 21, 29, 31)
- If N is a power of 2, the Fast Fourier Transform (FFT) is a computationally efficient method for calculating the DFT
- Often N=4096, 65,536, …
- FFT is available in Matlab and as subroutines for C++

Spectral Characterization

Key theorem useful for spectral characterization

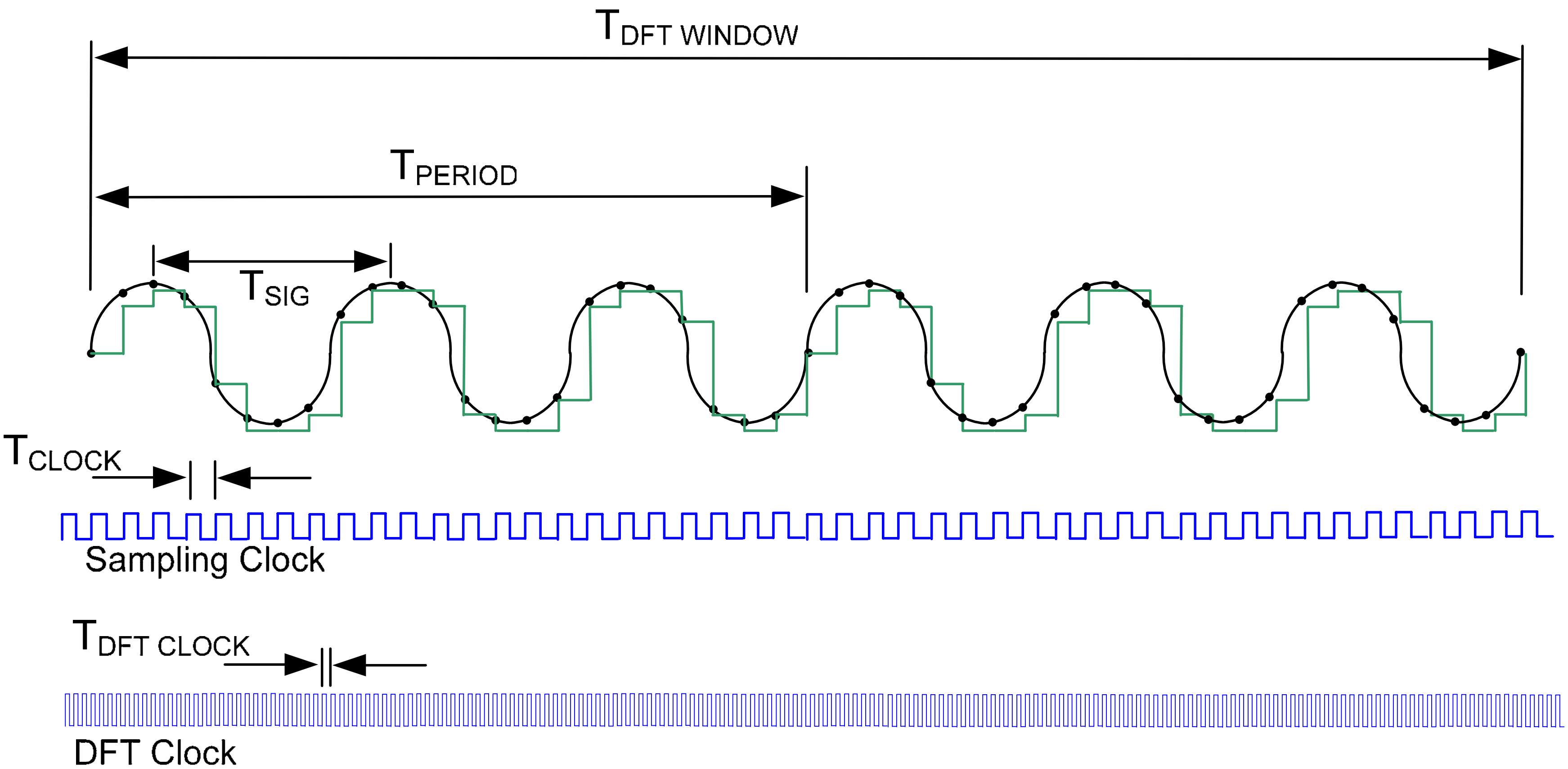
Theorem: If a periodic signal x(t) with period T=1/f is band-limited to frequency hf and if the signal is sampled N times over an integral number of periods, N_P, then

$$\left|A_m\right| = \frac{2}{N}\left|X\left(mN_P+1\right)\right| \qquad \textbf{for } 0 \le m \le h\text{-}1$$

where $\left\langle X\left(k\right)\right\rangle_{k=1}^{N-1}$ **is the DFT of the sampled sequence** $\left\langle x\left(kT_S\right)\right\rangle_{k=1}^{N-1}$ **where T_S is the sampling period.**

A_0, A_1, A_2, A_3, … are the magnitudes of the DFT elements X(0), $X(N_P+1)$, $X(2N_P+1)$, $X(3N_P+1)$, … respectively

Spectral Characterization



Spectral Characterization